

# The Logic of Anaphora Resolution<sup>1</sup>

David Beaver  
Stanford University

## Abstract

This paper concerns the semantics/pragmatics interface for natural language, and in particular the question of how anaphora resolution should be orchestrated in a dynamic semantics. Previous dynamic systems such as DPL have relied on preindexation of anaphors and antecedents. It is argued that this represents a serious inadequacy. A dynamic semantic system, RPL, is proposed which eliminates the need for preindexation, by combining an “Amsterdam-style” dynamic semantics with a pragmatic module. The semantics uses a novel extension of DMPL information states, in which ambiguity of anaphors is represented using multiple referent systems. It is argued that the resulting system not only provides a marked empirical improvement over dynamic predecessors, but also provides a quite general approach to the semantics/pragmatics interface.

## 1 Introduction

There is a bizarre assumption made in Groenendijk and Stokhof’s Dynamic Predicate Logic [GS91a] and Dynamic Montague Grammar [GS91b], and Heim’s File Change Semantics [He82], an assumption that is not made in Kamp’s Discourse Representation Theory [KR93]. This is the assumption that anaphor-antecedent relationships are presented ‘gift-wrapped’ to the semantics in the form of pre-indexed NPs.<sup>2</sup> Who or what is supposed to do the wrapping is unclear.

Perhaps syntax does the job of pre-indexation? Yet syntax seems unlikely to do more than provide a few extra local constraints on resolution via C-command conditions etc. If not syntax, then presumably pragmatics must do the job. But this would not sit well with what appears to be the architecture of DPL, DMG and FCS. For it seems that pragmatics should either operate after semantic interpretation, as is assumed e.g. in classic work on pragmatics by Grice, Austin and Searle, or at least should operate in tandem with semantics.

In [Be99], a general approach to ambiguity resolution in Dynamic Semantics is proposed. Here I exemplify this approach in the domain of anaphora resolution, by showing how a dynamic semantics can be formally combined with a pragmatic resolution module. For the purposes of this extended abstract, I use a naive form of parallelism for resolution.<sup>3</sup> The result dispenses with the indexing assumption, so that indexation can no longer be seen as a point dividing DRT

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<sup>1</sup>This extended abstract draws on material in [Be99], but extends that work formally. Thanks in particular to Maria Aloni and Paul Dekker.

<sup>2</sup>In this respect Musken’s “Compositional DRT” [Mu96] is not like DRT but like DPL/DMG/FCS, since it does assume pre-indexation.

<sup>3</sup>In a longer version of the paper, available on request, I show how the Centering model [GJW95] can be substituted.

from other dynamic systems. In fact RPL provides a model of resolution empirically superior to standard DRT, since RPL incorporates pragmatic constraints on resolution which are not found in standard DRT<sup>4</sup>.

The system developed is predicate logical, but involves non-standard operators and conventions. The semantics uses a notion of information state from DMPL [GSV95], but adapts the *referent systems* used there to entirely new purposes. Referent systems mediate between syntactic variables (henceforth *referents*) and discourse entities (for Groenendijk et al, *pegs*). In DMPL, as in the earlier [Ve94], a referent system is used to prevent requantification over a variable from destroying information about the discourse entity that the variable picks out. In RPL, while this role remains, referent systems have two additional functions. First, they represent uncertainty about which discourse entity a given syntactic variable refers to, and, second, they perform a book-keeping role, maintaining information about the grammatical role (e.g. subject) of the last NP to refer to each discourse entity. To represent uncertainty, multiple referent systems are allowed in a single information state, a possibility not found in [GSV95], and which necessitates a modified notion of entailment.

## 2 Semantics

The semantics for RPL is similar to that for DMPL. There are two new operators:  $\rho x$  introduces the referent  $x$  as a pronoun, which must be interpreted as picking out the same discourse entity as some previous referent, and  $[\phi]$  is the resolution of  $\phi$ .

**Definition D1 (Models, Information States)** (i) Models are pairs  $\langle W, D \rangle$  of a non-empty set of worlds and a non-empty domain of individuals. (ii) A world is a function from individual constants to members of  $D$ , and from  $n$ -ary predicate constants to sets of  $n$ -tuples of elements of  $D$ . (iii) An assignment function  $g$  is a finite sequence of members of  $D$ . For an  $n$ -member sequence,  $n$  is the domain of the function. For a positive integer  $r \leq n$ ,  $g(r)$  is the  $r$ -th member of the sequence. (iv) A referent system  $r$  is a function from referents to integers (these integers being understood as members of an associated assignment function.). (v) An information state  $s$  is a set of possibilities defined relative to some model,  $\langle W, D \rangle$ . A possibility  $i$  is a triple  $i = \langle r, g, w \rangle$  of a referent system, an assignment function and a world, such that the range of  $r$  is within the domain of  $g$ , the range of  $g$  is within the domain of the model,  $D$ , and  $w \in W$ . The minimal information state  $\odot$  is  $\emptyset \times \emptyset \times W$ , where  $\emptyset$  is a function with zero domain.

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<sup>4</sup>Similar in spirit to the present enterprise is Roberts' proposal in [Ro98], which discusses how a sophisticated pragmatic theory might be combined with DRT. The strength of her theory, its sophistication, is also a weakness, in the sense that Roberts relies upon an incompletely specified theory of text structure and communicative intention. In that sense, the proposal in the current paper is less ambitious, but more strongly predictive.

**Definition D2 (Semantics: basic clauses)**

$$\begin{aligned}
s[Rt_1 \dots t_n] &= \{i \in s \mid \langle i(t_1), \dots, i(t_n) \rangle \in i(R)\} \\
&\text{and is undefined if any component } i(\alpha) \text{ is undefined.} \\
\langle r, g, w \rangle(\alpha) &= w(\alpha) \text{ for } \alpha \text{ an individual or predicate constant} \\
&g(r(\alpha)) \text{ for } \alpha \text{ a referent in the domain of } r \\
&\text{and otherwise is undefined.} \\
s[t_1 = t_2] &= \{i \in s \mid i(t_1) = i(t_2)\} \\
s[\neg\phi] &= \{i \in s \mid i \text{ does not subsist in } s[\phi]\} \\
s[\phi \wedge \psi] &= s[\phi][\psi] \\
s[\exists x\phi] &= \bigcup_{d \in D} (s[x/d][\phi])
\end{aligned}$$

Two ways of adding referents are now defined. The notation  $s[x/d]$  is familiar from DMPL, with one slight modification.  $s[x/d]$  means the variant of  $s$  in which the referent  $x$  is mapped to a new discourse entity, which in turn all assignment functions map to  $d$ . Whereas in DMPL the discourse entity that  $x$  is mapped to is determined in terms of the range of the referent systems in  $s$ , below I use instead the domain of the assignment functions in  $s$ .<sup>5</sup> The notation  $s[x \downarrow n]$  is used to denote the variant of  $s$  in which all referent systems have been modified so that the referent  $x$  is mapped to the discourse entity  $n$ , which should be in the domain of assignment functions in  $s$ .

**Definition D3 (Referent setting, extension)**

Let  $i = \langle r, g, w \rangle \in I$ ;  $i' = \langle r', g', w' \rangle \in I$ ;  $n$  is the largest integer in the domain of  $g$ ;  $d \in D, s \in S$ .

$$\begin{aligned}
s[x/d] &= \{i[x/d] \mid i \in s\} \\
i[x/d] &= \langle r[x/(n+1)], g[(n+1)/d], w \rangle \\
i \leq i' &\text{ iff } g \subseteq g', \text{ \& } w = w' \\
i \text{ subsists in } s &\text{ iff } \exists i' \in s \text{ } i \leq i' \\
s[x \downarrow n] &= \{i[x \downarrow n] \mid i \in s\} \\
i[x \downarrow n] &= \langle r[x/n], g, w \rangle \\
d\text{-entities}(s) &= \{n \mid \exists \langle r, g, w \rangle \in s \text{ } n \in \text{dom}(g)\}
\end{aligned}$$

An operator is now defined which introduces a new referent, but leaves it indeterminate between all the discourse entities which have been introduced previously:

**Definition D4 (Anaphoric operator)**

$$s[\rho x]s' \text{ iff } s' = \bigcup_{n \in d\text{-entities}(s)} s[x \downarrow n]$$

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<sup>5</sup>The reason for this is that referent systems will occasionally be reset, losing information about which referents have been introduced.

**Definition D5 (Entailment, Possible Entailment)**

$$\begin{aligned}
\phi \models \psi & \quad \text{iff} \quad \odot[\phi] \leq \odot[\phi][\psi] \\
\text{resolutions}(s) & = \{t \mid \exists r \, t = \{\langle r', g, w \rangle \in s \mid r = r'\} \neq \emptyset\} \\
\phi \rightsquigarrow \psi & \quad \text{iff} \quad \exists s \in \text{resolutions}(\odot[\phi]) \rightarrow \exists t \in \text{resolutions}(s[\psi]) \, s \leq t
\end{aligned}$$

In the above,  $\rightsquigarrow$  should be read as “possibly entails”, i.e. there is a way of resolving  $\rho$ -referents such that the antecedent entails the consequent, although there may be other ways of resolving referents so that this is not the case.

Consider the argument: “Jane likes Mary. Therefore, she likes someone.” We translate this as the following valid argument pattern:  $\exists x \, x = j \wedge \exists y \, y = m \wedge \text{likes}(x, y) \rightsquigarrow \rho u \wedge \exists v \, \text{likes}(u, v)$

### 3 Ambiguity and Anaphoric Resolution

*Transition Preference Pragmatics*, introduced in [Be99] is a general approach to the semantic-pragmatics interface in a dynamic setting. The proposal is that compositional derivation fixes a set of possible state transitions, and that pragmatics (“to a first approximation”) merely provides a preference ordering over the alternative transitions. This approach will now be exemplified with respect to RPL.

Let us require that referents be integers. We will then assume a convention whereby in the translation of sentences of English to RPL, the integers are chosen according to some algorithm based on grammatical obliqueness, so that e.g. the subject of a main clause is always translated using the referent 1.

Formulae of RPL, if they involve  $\rho$ -formulae, can lead to unresolved states, in which it is uncertain which discourse object is referred to by a given referent. The output of such a formula can be split up into alternative *resolutions*, as we have seen. Suppose that for some input  $s$ , there is a set of alternative resolutions  $\tau$ . How can we select the *best* member of  $\tau$ ? That is the job which a pragmatic theory must be able to answer. But here, rather than delving into the question of which is the right pragmatic theory, I propose a token theory to show how in principle the job can be done.

The token theory can be stated as follows: (1) transitions will be ranked according to the degree of parallelism between input and output; (2) parallelism will be measured by the number of referents in the output which are mapped onto the same discourse object as the corresponding referent in the input; (3) a preference will be assumed for parallelism of referents introduced by the syntactically least oblique NPs. Thus we will prefer transitions that maintain subject parallelism, but if that fails to decide, we will prefer transitions providing direct object parallelism, and so on. This is the basis of the *max* predicate, introduced below:  $\text{max}_s(\tau)$  picks out the subset of resolutions in  $\tau$  which are most strongly parallel with  $s$ .

**Definition D6 (Maximal Transitions)**

$$\begin{aligned}\mathcal{R}(s) &= r \text{ iff } \forall \langle r', g, w \rangle \in s \ r = r' \\ \max_s(\tau) &= \{t \in \tau \mid \neg \exists u \in \tau \ \exists n \ \mathcal{R}(s)(n) = \mathcal{R}(u)(n) \neq \mathcal{R}(t)(n) \ \& \\ &\quad \forall n' < n \ (\mathcal{R}(s)(n') = \mathcal{R}(t)(n') \Leftrightarrow \mathcal{R}(s)(n') = \mathcal{R}(u)(n'))\}\end{aligned}$$

I now define a resolution operator  $[\cdot]$  which utilizes the definition of maximal transitions. The input,  $s$ , of  $[\phi]$  should be the output of a previous formula. A variant of  $s$  with all component referent systems emptied of information (the effect of “*clean*”) is updated with  $\phi$  to produce an intermediary state which may involve unresolvedness. The predicate *resolutions* breaks this state up into alternative fully resolved outputs, and the predicate *max* selects between these. To allow for the possibility of multiple maximal states, the union of maximal states is taken to produce the final output. In most cases this will be a fully resolved state, and will be the output which involves maximal parallelism with the input.

**Definition D7 (Resolution Operator)**

$$\begin{aligned}\text{clean}(s) &= \{\langle \emptyset, g, w \rangle \mid \exists r \langle r, g, w \rangle \in s\} \\ s[[\phi]] &= \cup \max_s(\text{resolutions}(\text{clean}(s)[[\phi]]))\end{aligned}$$

In the example below, the parallelism constraint favors outputs of the second sentence of (A) which map referent 1 onto the same discourse entity as in the outputs of the first sentence. It is thus an easily demonstrated formal property of the system defined that the translation of the discourse in (A) entails that of the sentence in (B).

- (A) A soldier meets a sailor. She likes her.  
 $\exists 1 \text{ soldier}(1) \wedge \exists 2 \text{ sailor}(2) \wedge \text{meets}(1, 2)$   
 $\wedge [\rho 1 \wedge \rho 2 \wedge \text{likes}(1, 2)]$
- (B) A soldier likes a sailor.  
 $\exists 1 \text{ soldier}(1) \wedge \exists 2 \text{ sailor}(2) \wedge \text{likes}(1, 2)$

## 4 Discussion

RPL is a methodological stepping stone on the way to a fragment of natural language. The latter could be achieved using an embedding into type theory, following the approach pioneered in [Mu96], although this goes beyond the scope of the current paper.

The RPL approach to dynamic ambiguity resolution is quite general. RPL itself will make correct predictions about anaphora only to the extent which the parallelism analysis itself makes correct predictions. But in principle, any theory of resolution which could be expressed as a preference ordering over alternative

transitions could be used instead of parallelism, including theories which defined only a partial ordering, and thus left pronoun reference underspecified.

Finally, anaphora resolution is only one aspect of the semantics/pragmatics interface. The current approach should generalize to other areas of ambiguity resolution and to presupposition accommodation. Indeed, the latter phenomenon has already been analyzed using orderings related to those used here in [Be95], and it remains for future work to combine these approaches.

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