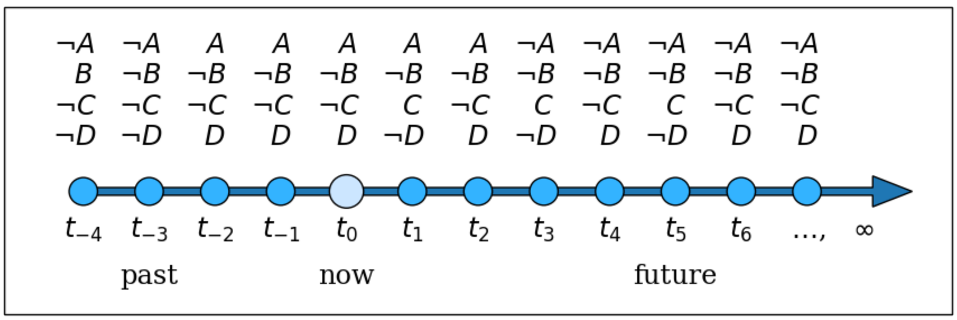
An Exercise on Logical Representations of Time

In this exercise we look at temporal representation from three different perspectives:

- Part 1 looks at how a tense logic model (an ordered sequence of propositional logic models) determines the truth of propositional formulae with tense operators.
- Part 2 is about using the representation to represent actual temporal information as expressed in English sentences.
  (Some of you may prefer to do Part 2 first. In fact it may be a good idea to look at both Parts 1 and 2 and then go back and forth between the two answering the specific questions of each part.)
- Part 3 considers the use of a 1st-order temporal logic, which provides a different way of representing temporal information.

1 Tense Logic Models

Consider the Tense Logic model illustrated below. Here time starts at a specific point t_{-4} (the big bang) and goes infinitely into the future. But once t_6 is reached the same state continues infinitely into the future.[1]



For each of the following formulae, determine whether it is True or False in the model. A formula should be considered True if it is true 'now', i.e. at the world state of t_0.

1. $A \rightarrow B$
2. $A \wedge \mathbf{P}\neg A \wedge \mathbf{F}\neg A$
3. $\mathbf{P}B \wedge \mathbf{GP}B$
4. $\mathbf{HP}B$
5. $\mathbf{G}(C \vee D)$
6. $\mathbf{F}C \wedge \mathbf{F}D$
7. $\mathbf{F}(C \wedge D)$
8. $\mathbf{P}A \rightarrow \mathbf{G}(C \vee D)$

Meaning of the tense operators:

$\mathbf{F}\phi$	'ϕ is true at some future time.'
$\mathbf{P}\phi$	'ϕ is true at some past time.'
$\mathbf{G}\phi$	'ϕ is true at all future times.'
$\mathbf{H}\phi$	'ϕ is true at all past times.'

9. $\mathbf{F}A \wedge \mathbf{FF}A$
10. $\mathbf{FFF}A$
11. $\mathbf{H}((A \wedge \neg B) \rightarrow D)$
12. $\mathbf{G}(B \rightarrow D)$

[1] Does it make sense to say that time passes on when nothing changes? In this general kind of model this is possible, since a state could be exactly the same as the previous state; but we could place restrictions on allowed models to take into account other conditions we may want to capture, regarding the nature of time. But in current example where, we only specify four propositions, we may suppose that although A, B, C and D may stay the same from one state to the next, other facts about the world could be changing.

2 Representation in Tense Logic

(a) Translate each of the following sentences into tense logic:

1. I have never eaten a snail.
2. I will only eat a haggis if I am in Scotland.
3. If you have not bought a ticket you will not be able to go to the concert.
4. I have eaten broccoli and chocolate but never at the same time.
5. I will visit Paris and then after that I will visit Rome.
6. Leeds was always beautiful, except when it was raining.
7. I shall not go out before the baby sitter arrives.
8. If I win the lottery I will buy a helicopter.
9. I've not seen her since before I first met you.
10. You will always be lucky or unlucky but in either case your luck will change.

(b) Give a natural language sentence corresponding to the tense logic formula

$$\mathbf{P}\chi \rightarrow \mathbf{G}(\mathbf{F}\chi \vee \neg\xi),$$

where χ represents the proposition 'She eats chocolate' and ξ represents the proposition 'She is alive'.

(c) In tense logic *define* a new construct $\mathbf{A}\phi$, which means there is no time (past, present or future) when ϕ is false — i.e. ϕ is always true. Your definition should be of the form $\mathbf{A}\phi \equiv_{def} \Phi(\phi)$, where $\Phi(\phi)$ should be replaced by some formula containing only the standard propositional operators, the standard tense operators and one or more occurrences of ϕ.

3 A 1st-Order Temporal Representation Language

Let $\mathcal{TL}$ be a 1st-order temporal language containing time variables, $t_1, \ldots, t_n$, a strict temporal ordering relation $<$, a constant 'now' denoting the current time and a construct $\mathsf{HoldsAt}(\phi, t)$ meaning that ϕ is true at time t.

1. Use $\mathcal{TL}$ to give a formula which means the same as the tense logic formula $\mathbf{F}A$.

2. Use $\mathcal{TL}$ to represent the sentence 'It will not rain before it gets cloudy.'

3. Explain concisely the meaning of the $\mathcal{TL}$ formula

$$\forall t_1 \forall t_2[(t_1 < t_2) \rightarrow \exists t_3[(t_1 < t_3) \wedge (t_3 < t_2)] \ .$$

4. Is propositional tense logic more or less *expressive* than $\mathcal{TL}$?