Axioms and Models Exercise

Owners and Pets: a question of responsibility

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When a family adopts a new pet, it is important to make clear which member(s) of the family will responsible for that pet, namely it is important to define owner-pet relations. The following symbols and formulae describe the possible structures of a family composed of humans and pets, and some possible relations of ownership between them. (Can you find a situation that a responsible family would not allow to happen?).

Study axioms and then do the exercises below.

Symbols used:

- $O(x)$ - x is an owner
- $P(x)$ - x is a pet.
- $R(x, y)$ - x is the owner of y.

Axioms:

1. $\forall x(O(x) \vee P(x))$.
2. $\neg\exists x(O(x) \wedge P(x))$.
3. $\exists x(P(x))$.
4. $\forall x\forall y\forall z((P(x) \wedge P(y) \wedge P(z)) \rightarrow (x = y \vee x = z \vee y = z))$.
5. $\exists x\exists y(O(x) \wedge O(y) \wedge \neg(x = y))$.
6. $\forall x\forall y\forall z((O(x) \wedge O(y) \wedge O(z)) \rightarrow (x = y \vee x = z \vee y = z))$.
7. $\forall x\forall y(xRy \rightarrow (O(x) \wedge P(y)))$.
8. $\forall x(O(x) \rightarrow (\exists y(xRy)))$.

(a) Translate each of the axioms 1–8 into an English sentence, in accordance with the given interpretation.

(b) Using a graphical representation (where nodes are entities, properties are labels next to the nodes and relations are arrows between nodes), draw diagrams representing all possible models of the 8 axioms. Each diagram you draw should satisfy all of the axioms

(c) Create a Python file `owners_and_pets_models.py`. In the file give a Python representation of all the models of the axioms, using the Python format illustrated on the next page. This file can be submitted on Gradescope by selecting the link for the "Models Exercise — Pets (formative)" assignment. It will check all your models and give a score for your submission. This is just for your feedback. It does not count towards your module grade.

```
## Filename: owners_and_pets_models.py

MODELS = [
   {
    'DOMAIN' : {1,2,3},
    'Owner'  : {1,2},
    'Pet'    : {2},
    'R'      : { (1,1), (1,3), (2,3) }
   },
   ## Add more models specified in similar form to the above,
   ## and remove the above, since it does not satisfy the axioms.
  ]
```