Module Title: Knowledge Representation and Reasoning

School of Computing

Semester 1 2019/20

Calculator instructions:

- You **are** allowed to use a non-programmable calculator only from the following list of approved models in this examination: **Casio FX-82**, **Casio FX-83**, **Casio FX-85**.

Dictionary instructions:

- A basic English dictionary is available to use: raise your hand and ask an invigilator, if you need it.

Examination Information

- There are **5** pages to this examination.
- There are **2 hours** to complete the examination.
- Answer **all 3** questions.
- The number in brackets **[ ]** indicates the marks available for each question or part question.
- You are reminded of the need for clear presentation in your answers.
- The total number of marks for this examination paper is **60**.
- You are allowed to use annotated materials.

Please do not remove this paper from the exam venue.

Question 1

(a) Translate the following sentence into *Propositional Logic*: **[2 marks]**

I go to the park on Sundays unless it is raining.

(b) Translate the following sentences into *First-Order Predicate Logic* (using equality where necessary):

(i) John found a green beetle. **[2 marks]**

(ii) Rabbits don't like chocolate, except for some old rabbits. **[2 marks]**

(iii) All friends of Mary know each other. **[2 marks]**

(iv) One of my uncles does not like any of his own children. **[2 marks]**

(c) Give an English sentence that captures the meaning of the following formula of first-order logic in a concise and natural way: **[2 marks]**

$$\begin{aligned}&\forall x[\ \text{Bicycle}(x) \rightarrow \\ &\quad \exists y \exists z[\ \neg(y = z) \wedge \text{Wheel}(y) \wedge \text{Wheel}(z) \wedge \\ &\qquad \text{HasPart}(x, y) \wedge \text{HasPart}(x, z) \wedge \\ &\qquad \forall w[(\text{HasPart}(x, w) \wedge \text{Wheel}(w)) \rightarrow (w = y \vee w = z)]\ ]\ ]\end{aligned}$$

(d) $\mathcal{M} = \langle \mathcal{D}, \delta \rangle$ is a model for a first-order language with two unary predicates P and Q and a binary relation predicate R. The domain of $\mathcal{M}$ is the set $\{a, b, c, d, e\}$, and the denotation of the predicates is:

- $\delta(P) = \{a, b, c\}$
- $\delta(Q) = \{c, d, e\}$
- $\delta(R) = \{\langle a, d \rangle, \langle b, e \rangle, \langle c, c \rangle\}$

Which of the following formulae are satisfied by this model? **[2 marks]**

F1. $\exists x[P(x) \wedge Q(x)]$

F2. $\exists x \exists y[\neg(x = y) \wedge \forall z[\neg R(z, x) \wedge \neg R(z, y)]]$

F3. $\forall x[P(x) \rightarrow \exists y[R(x, y) \wedge Q(y)]]$

F4. $\neg \exists x \exists y[Q(x) \wedge Q(y) \wedge R(x, y)]$

(e) Use the *Sequent Calculus* to show that the following sequent is valid: **[6 marks]**

$$\forall x[P(x) \wedge Q(x)],\ (P(a) \wedge Q(b)) \rightarrow R \ \vdash\ R$$

You should only use rules from the following rule set, which was presented in the lecture slides, to construct your proof:

$$\frac{\textit{Axiom}}{\alpha,\ \Gamma\ \vdash\ \alpha,\ \Delta}$$

$$\frac{\alpha,\ \beta,\ \Gamma\ \vdash\ \Delta}{(\alpha \wedge \beta),\ \Gamma\ \vdash\ \Delta}\,[\wedge\vdash] \qquad \frac{\Gamma \vdash\ \alpha,\ \Delta \quad \textit{and} \quad \Gamma \vdash \beta, \Delta}{\Gamma\ \vdash (\alpha \wedge \beta),\ \Delta}\,[\vdash\wedge]$$

$$\frac{\alpha, \Gamma\ \vdash\ \Delta \quad \textit{and} \quad \beta, \Gamma\ \vdash\ \Delta}{(\alpha \vee \beta),\ \Gamma\ \vdash\ \Delta}\,[\vee\vdash] \qquad \frac{\Gamma\ \vdash\ \alpha,\ \beta,\ \Delta}{\Gamma\ \vdash\ (\alpha\ \vee\ \beta),\ \Delta}\,[\vdash\ \vee]$$

$$\frac{\Gamma\ \vdash\ \alpha,\ \Delta}{\neg\alpha,\ \Gamma\ \vdash\ \Delta}\,[\neg\vdash] \qquad \frac{\Gamma,\ \alpha\ \vdash\ \Delta}{\Gamma\ \vdash\ \neg\alpha,\ \Delta}\,[\vdash\neg]$$

$$\frac{\Gamma,\ \neg\alpha \vee \beta\ \vdash\ \Delta}{\Gamma,\ \alpha\ \rightarrow\ \beta\ \vdash\ \Delta}\,[\rightarrow\ \vdash r.w.] \qquad \frac{\Gamma\ \vdash\ \neg\alpha \vee \beta,\ \Delta}{\Gamma\ \vdash\ \alpha\ \rightarrow\ \beta,\ \Delta}\,[\vdash\ \rightarrow r.w.]$$

$$\frac{\forall x[\Phi(x)],\ \Phi(k),\ \Gamma\ \vdash\ \Delta}{\forall x[\Phi(x)],\ \Gamma\ \vdash\ \Delta}\,[\forall\vdash] \qquad \frac{\Gamma\ \vdash\ \Phi(k), \Delta}{\Gamma\ \vdash\ \forall x[\Phi(x)],\ \Delta}\,[\vdash\forall]^{\dagger}$$

† where κ cannot occur anywhere in the lower sequent.

[Question 1 total: 20 marks]

Question 2

(a) (i) Give the set of *clausal* formulae (i.e. formulae in *conjunctive normal form*) corresponding to the following propositional formulae: **[4 marks]**

$$\neg\neg P,\ (P \to Q),\ ((Q \wedge R) \to S),\ \neg(R \to S)$$

(ii) Give a proof that these formulae are inconsistent using *binary propositional resolution*. **[4 marks]**

(b) Translate the following sentence into *Propositional Tense Logic*: **[2 marks]**

I found your wallet after you had left.

(c) An AI specialist wants to model using *Situation Calculus* a situation in which a group of agents each possess various precious jewels of different colours, which they exchange with each other. Each agent wants to acquire any jewel that is of a colour that they do not possess, and, if they only have one jewel of a particular colour, they want to keep that jewel. Two agents will exchange jewels with each other only if: one of the two receives a jewel that they want to acquire; and neither of them gives a jewel that they want to keep.

The only *fluent* used in the representation will be:

- has(a, j) — agent a has jewel j.

The following predicate will also be used:

- Colour(j, c) — jewel j is of colour c.

(This is a normal predicate, not a fluent, since the colour of a jewel cannot change.)

The crucial part of the Situation Calculus representation will be axiomatisation of the action of two agents exchanging jewels. This action will be represented as follows:

- $\mathbf{Exchange}(a_1, j_1, a_2, j_2)$ — agent a_1 gives jewel j_1 to agent a_2, and agent a_2 gives jewel j_2 to agent a_1.

(i) Give an *effect* axiom specifying all changes of fluents caused by the $\mathbf{Exchange}$ action. **[3 marks]**

(ii) Give an appropriate *pre-condition* axiom for the $\mathbf{Exchange}$ action, which captures the conditions required for an exchange to take place. **[4 marks]**

(iii) Give a suitable *frame axiom* which specifies what will stay the same after the $\mathbf{Exchange}$ action. **[3 marks]**

[Question 2 total: 20 marks]

Question 3

(a) For each of the following *Prolog* queries, give the value of the variable `X` after the query has been executed:

(i) `?- Y = 8, Y/2 = X.` **[1 mark]**

(ii) `?- [ _ | Y] = [which, one, is, it], [ _, X | _] = Y.` **[1 mark]**

(iii) `?- L = [1,2], setof( [A,B], (member(A,L), member(B,L)), X).` **[2 marks]**

(b) Consider the following formulae involving topological relations of the *Region Connection Calculus* (RCC) and the *convex hull* function, conv. The constants (a, b and c) refer to particular spatial regions. In each case, draw a configuration of the regions referred to by these constants that satisfies the formula, labelling your diagram to indicate which region is which:

(i) $\mathsf{NTPP}(\mathsf{sum}(\mathsf{a},\mathsf{b}),\mathsf{c}) \wedge \mathsf{PO}(\mathsf{a},\mathsf{b})$ **[2 marks]**

(ii) $\mathsf{P}(\mathsf{a},\mathsf{b}) \wedge \mathsf{P}(\mathsf{b},\mathsf{c}) \wedge \mathsf{TPP}(\mathsf{a},\mathsf{c})$ **[2 marks]**

(iii) $\mathsf{DC}(\mathsf{a},\mathsf{b}) \wedge \mathsf{PO}(\mathsf{a},\mathsf{conv}(\mathsf{b}))$ **[2 marks]**

(c) Represent the following statements in *Description Logic*:

(i) Humans are a kind of non-aquatic mammal. **[2 marks]**

(ii) Every happy child has a happy friend. **[2 marks]**

(d) Give an explanation in English of the following *Default Logic* rule: **[2 marks]**

$$\mathsf{Friend}(x,y) \wedge \mathsf{Friend}(x,z) \ : \ \mathsf{Friend}(y,z) \ / \ \mathsf{Friend}(y,z)$$

(e) This question concerns a *Fuzzy Logic* in which the following definitions of truth functions for *linguistic modifiers* are specified:

$$\mathsf{quite}(\phi) = \phi^{\frac{1}{2}} \qquad \mathsf{very}(\phi) = \phi^2$$

The logic is used to describe Jumbo the elephant, who possesses certain characteristics to the following degrees:

$$\mathsf{Large}(\mathsf{jumbo}) = 0.7 \qquad \mathsf{Intelligent}(\mathsf{jumbo}) = 0.36$$

Translate the following sentences into fuzzy logic and also give the fuzzy truth value of each proposition (under the standard fuzzy interpretation of the Boolean connectives):

(i) Jumbo is quite intelligent. **[2 marks]**

(ii) Jumbo is intelligent and not very large. **[2 marks]**

[Question 3 total: 20 marks]

[Grand total: 60 marks]