Module Title: Knowledge Representation and Reasoning

School of Computing

Semester 1 2019/20

Calculator instructions:

- You **are** allowed to use a non-programmable calculator only from the following list of approved models in this examination: **Casio FX-82**, **Casio FX-83**, **Casio FX-85**.

Dictionary instructions:

- A basic English dictionary is available to use: raise your hand and ask an invigilator, if you need it.

Examination Information

- There are **8** pages to this examination.
- There are **2 hours** to complete the examination.
- Answer **all 3** questions.
- The number in brackets **[ ]** indicates the marks available for each question or part question.
- You are reminded of the need for clear presentation in your answers.
- The total number of marks for this examination paper is **60**.
- You are allowed to use annotated materials.

Please do not remove this paper from the exam venue.

** With Solutions **

Question 1

(a) Translate the following sentence into *Propositional Logic*: **[2 marks]**

I go to the park on Sundays unless it is raining.

Answer: $\mathit{Sunday} \rightarrow (\mathit{Park} \leftrightarrow \neg \mathit{Raining})$.

Also allow: $(\mathit{Sunday} \wedge \neg \mathit{Raining}) \rightarrow \mathit{Park}$ or something equivalent to that.

(b) Translate the following sentences into *First-Order Predicate Logic* (using equality where necessary):

(i) John found a green beetle. **[2 marks]**
Answer: $\exists x[\mathsf{Found}(\mathsf{john}, x) \wedge \mathsf{Beetle}(x) \wedge \mathsf{Green}(x)]$

(ii) Rabbits don't like chocolate, except for some old rabbits. **[2 marks]**
Answer: $\forall x[(\mathsf{Rabbit}(x) \wedge \mathsf{Likes}(x, \mathsf{choc}) \rightarrow \mathsf{Old}(x)]$

(iii) All friends of Mary know each other. **[2 marks]**
Answer: $\forall x \forall y[(\mathsf{Friend}(x, \mathsf{mary}) \wedge \mathsf{Friend}(x, \mathsf{mary})) \rightarrow \mathsf{Knows}(x, y)\ ]$

(iv) One of my uncles does not like any of his own children. **[2 marks]**
Answer: $\exists x[\mathsf{UncleOf}(x, \mathsf{me}) \wedge \forall y[\mathsf{ChildOf}(y, x) \rightarrow \neg \mathsf{Likes}(x, y)]]$

(c) Give an English sentence that captures the meaning of the following formula of first-order logic in a concise and natural way: **[2 marks]**

$$\begin{aligned} &\forall x[\ \mathsf{Bicycle}(x) \rightarrow \\ &\quad \exists y \exists z[\ \neg(y = z) \wedge \mathsf{Wheel}(y) \wedge \mathsf{Wheel}(z) \wedge \\ &\qquad \mathsf{HasPart}(x, y) \wedge \mathsf{HasPart}(x, z) \wedge \\ &\qquad \forall w[(\mathsf{HasPart}(x, w) \wedge \mathsf{Wheel}(w)) \rightarrow (w = y \vee w = z)]\ ]\ ] \end{aligned}$$

Answer: Bicycles have two wheels. (1 mark for a sentence that is correct but long-winded and/or un-natural.)

(d) $\mathcal{M} = \langle \mathcal{D}, \delta \rangle$ is a model for a first-order language with two unary predicates P and Q and a binary relation predicate R. The domain of $\mathcal{M}$ is the set $\{a, b, c, d, e\}$, and the denotation of the predicates is:

- $\delta(P) = \{a, b, c\}$
- $\delta(Q) = \{c, d, e\}$
- $\delta(R) = \{\langle a, d \rangle, \langle b, e \rangle, \langle c, c \rangle\}$

Which of the following formulae are satisfied by this model? **[2 marks]**

F1. $\exists x[P(x) \wedge Q(x)]$

F2. $\exists x \exists y[\neg(x = y) \wedge \forall z[\neg R(z, x) \wedge \neg R(z, y)]]$

F3. $\forall x[P(x) \rightarrow \exists y[R(x, y) \wedge Q(y)]]$

F4. $\neg \exists x \exists y[Q(x) \wedge Q(y) \wedge R(x, y)]$

Answer: F1 yes, F2 yes, F3 yes, F4 no. (2 marks if exactly these given. 1 mark if one missed out or one extra given. 0, if two or more wrong, since this could be guesswork.)

(e) Use the *Sequent Calculus* to show that the following sequent is valid: **[6 marks]**

$$\forall x[P(x) \wedge Q(x)],\ (P(a) \wedge Q(b)) \rightarrow R \ \vdash\ R$$

You should only use rules from the following rule set, which was presented in the lecture slides, to construct your proof:

$$\dfrac{\textit{Axiom}}{\alpha,\ \Gamma\ \vdash\ \alpha,\ \Delta}$$

$$\dfrac{\alpha,\ \beta,\ \Gamma\ \vdash\ \Delta}{(\alpha \wedge \beta),\ \Gamma\ \vdash\ \Delta}\,[\wedge\vdash] \qquad \dfrac{\Gamma \vdash\ \alpha,\ \Delta \quad \textit{and} \quad \Gamma \vdash \beta, \Delta}{\Gamma\ \vdash (\alpha \wedge \beta),\ \Delta}\,[\vdash\wedge]$$

$$\dfrac{\alpha, \Gamma\ \vdash\ \Delta \quad \textit{and} \quad \beta, \Gamma\ \vdash\ \Delta}{(\alpha \vee \beta),\ \Gamma\ \vdash\ \Delta}\,[\vee\vdash] \qquad \dfrac{\Gamma\ \vdash\ \alpha,\ \beta,\ \Delta}{\Gamma\ \vdash\ (\alpha\ \vee\ \beta),\ \Delta}\,[\vdash\ \vee]$$

$$\dfrac{\Gamma\ \vdash\ \alpha,\ \Delta}{\neg\alpha,\ \Gamma\ \vdash\ \Delta}\,[\neg\vdash] \qquad \dfrac{\Gamma,\ \alpha\ \vdash\ \Delta}{\Gamma\ \vdash\ \neg\alpha,\ \Delta}\,[\vdash\neg]$$

$$\dfrac{\Gamma,\ \neg\alpha \vee \beta\ \vdash\ \Delta}{\Gamma,\ \alpha\ \rightarrow\ \beta\ \vdash\ \Delta}\,[\rightarrow\vdash r.w.] \qquad \dfrac{\Gamma\ \vdash\ \neg\alpha \vee \beta,\ \Delta}{\Gamma\ \vdash\ \alpha\ \rightarrow\ \beta,\ \Delta}\,[\vdash\rightarrow r.w.]$$

$$\dfrac{\forall x[\Phi(x)],\ \Phi(k),\ \Gamma\ \vdash\ \Delta}{\forall x[\Phi(x)],\ \Gamma\ \vdash\ \Delta}\,[\forall\vdash] \qquad \dfrac{\Gamma\ \vdash\ \Phi(k), \Delta}{\Gamma\ \vdash\ \forall x[\Phi(x)],\ \Delta}\,[\vdash\forall]^{\dagger}$$

† where κ cannot occur anywhere in the lower sequent.

Answer: The sequent is valid as shown by the following proof:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\textit{Axiom}}{\forall x[P(x) \wedge Q(x)],\ P(a),\ Q(a) \vdash\ P(a),\ R}}{\forall x[P(x) \wedge Q(x)],\ P(a) \wedge Q(a)\ \vdash\ P(a),\ R}\,[\wedge\vdash]}{\forall x[P(x) \wedge Q(x)]\ \vdash\ P(a),\ R}\,[\forall\vdash] \quad \dfrac{\dfrac{\dfrac{\textit{Axiom}}{\forall x[P(x) \wedge Q(x)],\ P(b),\ Q(b) \vdash\ Q(b),\ R}}{\forall x[P(x) \wedge Q(x)],\ P(b) \wedge Q(b)\ \vdash\ Q(b),\ R}\,[\wedge\vdash]}{\forall x[P(x) \wedge Q(x)]\ \vdash\ Q(b),\ R}\,[\forall\vdash]}{\forall x[P(x) \wedge Q(x)]\ \vdash\ (P(a) \wedge Q(b)),\ R}\,[\vdash\wedge]}{\forall x[P(x) \wedge Q(x)],\ \neg(P(a) \wedge Q(b))\ \vdash\ R}\,[\neg\vdash] \quad \dfrac{\textit{Axiom}}{\forall x[P(x) \wedge Q(x)],\ R\ \vdash\ R}}{\forall x[P(x) \wedge Q(x)],\ \neg(P(a) \wedge Q(b)) \vee R\ \vdash\ R}\,[\vee\vdash]}{\forall x[P(x) \wedge Q(x)],\ (P(a) \wedge Q(b)) \rightarrow R\ \vdash\ R}\,[\rightarrow\vdash \text{r.w.}]$$

There is basically a mark for each correct rule application. Some credit may be given for almost correct rule applications.

[Question 1 total: 20 marks]

Question 2

(a) (i) Give the set of *clausal* formulae (i.e. formulae in *conjunctive normal form*) corresponding to the following propositional formulae: **[4 marks]**

$$\neg\neg P,\ (P \rightarrow Q),\ ((Q \wedge R) \rightarrow S),\ \neg(R \rightarrow S)$$

Answer:

$$\{\{P\},\ \{\neg P, Q\},\ \{\neg Q, \neg R, S\},\ \{R\},\ \{\neg S\}\}$$

(ii) Give a proof that these formulae are inconsistent using *binary propositional resolution*. **[4 marks]**

Answer:

1. $\{P\}$
2. $\{\neg P, Q\}$
3. $\{\neg Q, \neg R, S\}$
4. $\{R\}$
5. $\{\neg S\}$
6. $\{Q\}$ 1+2
7. $\{\neg R, S\}$ 3+6
8. $\{S\}$ 4+7
9. $\emptyset$ 5+8

(b) Translate the following sentence into *Propositional Tense Logic*: **[2 marks]**

I found your wallet after you had left.

Answer: $\mathbf{P}(\textit{IFoundWallet} \wedge \mathbf{P}\ \textit{YouLeave})$

(c) An AI specialist wants to model using *Situation Calculus* a situation in which a group of agents each possess various precious jewels of different colours, which they exchange with each other. Each agent wants to acquire any jewel that is of a colour that they do not possess, and, if they only have one jewel of a particular colour, they want to keep that jewel. Two agents will exchange jewels with each other only if: one of the two receives a jewel that they want to acquire; and neither of them gives a jewel that they want to keep.

The only *fluent* used in the representation will be:

- has(a, j) — agent a has jewel j.

The following predicate will also be used:

- Colour(j, c) — jewel j is of colour c.

(This is a normal predicate, not a fluent, since the colour of a jewel cannot change.)

The crucial part of the Situation Calculus representation will be axiomatisation of the action of two agents exchanging jewels. This action will be represented as follows:

- $\textbf{Exchange}(a_1, j_1, a_2, j_2)$ — agent a_1 gives jewel j_1 to agent a_2, and agent a_2 gives jewel j_2 to agent a_1.

(i) Give an *effect* axiom specifying all changes of fluents caused by the $\textbf{Exchange}$ action. **[3 marks]**

Answer:

$$\begin{aligned}(\;& holds(\mathsf{has}(a_1, j_2), result(\textbf{Exchange}(a_1, j_1, a_2, j_2), s)) \wedge \\ & holds(\mathsf{has}(a_2, j_1), result(\textbf{Exchange}(a_1, j_1, a_2, j_2), s)) \wedge \\ & \neg holds(\mathsf{has}(a_1, j_1), result(\textbf{Exchange}(a_1, j_1, a_2, j_2), s)) \wedge \\ & \neg holds(\mathsf{has}(a_2, j_2), result(\textbf{Exchange}(a_1, j_1, a_2, j_2), s)) \\ )\; & \leftarrow \; poss(\; \textbf{Exchange}(a_1, i_1, a_2, i_2),\; s\;)\end{aligned}$$

Note: this describes the changes in the only fluent, has, after an **Exchange** action (on condition that the action is possible).

(ii) Give an appropriate *pre-condition* axiom for the $\textbf{Exchange}$ action, which captures the conditions required for an exchange to take place. **[4 marks]**

Answer: $poss(\; \textbf{Exchange}(a_1, i_1, a_2, i_2),\; s\;) \leftarrow$

\# agents must have the jewel they are giving

$holds(\mathsf{has}(a_1, i_1), s) \wedge holds(\mathsf{has}(a_2, i_2), s) \wedge$

\# the jewels have certain colours

$\mathsf{Colour}(j_1, c_1) \wedge \mathsf{Colour}(j_2, c_2) \wedge$

\# each agent has another jewel of the colour they are giving

$\exists j_3[\neg(j_3 = j_1) \wedge \mathsf{Colour}(j_3, c_1) \wedge holds(\mathsf{has}(a_1, j_3, s))] \wedge$

$\exists j_3[\neg(j_3 = j_2) \wedge \mathsf{Colour}(j_3, c_2) \wedge holds(\mathsf{has}(a_2, j_3, s))]) \wedge$

\# one of the agents does not have a jewel of the colour they are receiving

$(\neg\exists j_3[\mathsf{Colour}(j_3, c_1) \wedge holds(\mathsf{has}(a_2, j_3), s)]$

$\vee$

$\neg\exists j_3[\mathsf{Colour}(j_3, c_1) \wedge holds(\mathsf{has}(a_2, j_3), s)])$

(iii) Give a suitable *frame axiom* which specifies what will stay the same after the $\textbf{Exchange}$ action. **[3 marks]**

Answer: $holds(\mathsf{has}(a, j), result(\textbf{Exchange}(a_1, j_1, a_2, j_2), s))$

$$\leftrightarrow$$

$$(holds(\mathsf{has}(a, j), s) \wedge (\neg((a = a_1) \vee (a = a_2)) \vee \neg((j = j_1) \vee (j = j_2)))$$

Note: this says that has relations where either the agent or the jewel is not involved in the **Exchange** stay the same, before and after the **Exchange**. It is OK to have $\leftarrow$ instead of $\leftrightarrow$. The formula with $\leftarrow$ is the limited (but commonly used) form of the Frame axiom which applies when reasoning about what stays the same after an action takes place.

[Question 2 total: 20 marks]

Question 3

(a) For each of the following *Prolog* queries, give the value of the variable `X` after the query has been executed:

(i) `?- Y = 8, Y/2 = X.` **[1 mark]**

(ii) `?-  [ _ | Y] = [which, one, is, it], [ _, X | _] = Y.` **[1 mark]**

(iii) `?- L = [1,2], setof( [A,B], (member(A,L), member(B,L)), X).` **[2 marks]**

Answer:
`(i) 8/2`
`(ii) is`
`(iii) [[1,1],[1,2],[2,1],[2,2]] (Allow any ordering of the sublists.)`

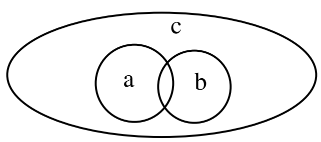
(b) Consider the following formulae involving topological relations of the *Region Connection Calculus* (RCC) and the *convex hull* function, conv. The constants (a, b and c) refer to particular spatial regions. In each case, draw a configuration of the regions referred to by these constants that satisfies the formula, labelling your diagram to indicate which region is which:

(i) $\mathsf{NTPP}(\mathsf{sum}(\mathsf{a},\mathsf{b}),\mathsf{c}) \wedge \mathsf{PO}(\mathsf{a},\mathsf{b})$ **[2 marks]**

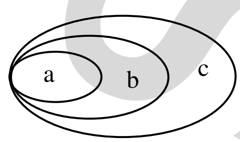
(ii) $\mathsf{P}(\mathsf{a},\mathsf{b}) \wedge \mathsf{P}(\mathsf{b},\mathsf{c}) \wedge \mathsf{TPP}(\mathsf{a},\mathsf{c})$ **[2 marks]**

(iii) $\mathsf{DC}(\mathsf{a},\mathsf{b}) \wedge \mathsf{PO}(\mathsf{a},\mathsf{conv}(\mathsf{b}))$ **[2 marks]**

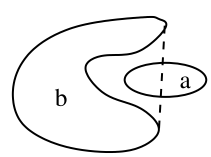
Answer: Possible diagrams are as follows:



(i)



(ii)



(iii)

(c) Represent the following statements in *Description Logic*:

(i) Humans are a kind of non-aquatic mammal. **[2 marks]**
Answer: $\mathsf{Human} \sqsubset (\neg\mathsf{Aquatic} \sqcap \mathsf{mammal})$

(ii) Every happy child has a happy friend. **[2 marks]**
Answer: $(\mathsf{Happy} \sqcap \mathsf{Child}) \sqsubset \exists\mathsf{hasFriend}.\mathsf{Happy}$

(d) Give an explanation in English of the following *Default Logic* rule: **[2 marks]**

$$\mathsf{Friend}(x,y) \wedge \mathsf{Friend}(x,z)\ :\ \mathsf{Friend}(y,z)\ /\ \mathsf{Friend}(y,z)$$

Answer: If someone has two friends then in the absence of information to the contrary one can assume the two are also friends with each other.

(e) This question concerns a *Fuzzy Logic* in which the following definitions of truth functions for *linguistic modifiers* are specified:

$$\text{quite}(\phi) = \phi^{\frac{1}{2}} \qquad \text{very}(\phi) = \phi^2$$

The logic is used to describe Jumbo the elephant, who possesses certain characteristics to the following degrees:

$$\text{Large(jumbo)} = 0.7 \qquad \text{Intelligent(jumbo)} = 0.36$$

Translate the following sentences into fuzzy logic and also give the fuzzy truth value of each proposition (under the standard fuzzy interpretation of the Boolean connectives):

(i) Jumbo is quite intelligent. **[2 marks]**

(ii) Jumbo is intelligent and not very large. **[2 marks]**

Answer:

A) quite(Intelligent(jumbo)) (1 mark)

Truth value $(0.36)^{1/2} = 0.6$ (1 mark)

B) Intelligent(jumbo))) ∧ ¬very(Large(jumbo))) (1 mark)

Truth value $= Min(0.36, 1 - (0.7)^2) = Min(0.36, 0.51) = 0.36$ (1 mark)

[Question 3 total: 20 marks]

[Grand total: 60 marks]