This question paper consists of 8 printed pages, each of which is identified by the Code Number COMP5450M.

A non-programmable calculator may be used.
Answer All Questions.
This is an open book examination.
Any written or printed material is permitted.

School of Computing

January 2019

COMP5450M

KNOWLEDGE REPRESENTATION AND REASONING (MSc)

Time allowed: 2 hours

PLEASE DO NOT REMOVE THIS PAPER FROM THE EXAM ROOM

Answer ALL THREE questions

The marks available for each part of each question are clearly indicated.

Question 1

(a) Translate the following sentence into *Propositional Logic*:

- I go shopping on Mondays and Tuesdays. **[2 marks]**
  Answer: $(Monday \lor Tuesday) \rightarrow Shopping$.

(b) Translate the following sentences into *First-Order Predicate Logic* (using equality where necessary):

(i) Some yellow frogs are poisonous. **[2 marks]**
Answer: $\exists x[\mathsf{Frog}(x) \land \mathsf{Yellow}(x) \land \mathsf{Poisonous}(x)]$

(ii) All Helen's rabbits are white or grey. **[2 marks]**
Answer: $\forall x[(\mathsf{Rabbit}(x) \land \mathsf{Owns}(\mathsf{helen}, x)) \rightarrow (\mathsf{White}(x) \lor \mathsf{Grey}(x))]$

(iii) No dog ate more than one biscuit. **[2 marks]**
Answer: $\neg\exists x \exists y \exists z[\,\mathsf{Dog}(x) \land \mathsf{Biscuit}(y) \land \mathsf{Biscuit}(z) \land \neg(y = z) \land \mathsf{Ate}(x, y) \land \mathsf{Ate}(x, z)\,]$

(iv) Edward hates everyone except himself. **[2 marks]**
Answer: $\forall x[\mathsf{Hates}(\mathsf{ed}, x) \leftrightarrow \neg(x = \mathsf{ed})]$

(c) $\mathcal{M} = \langle \mathcal{D}, \delta \rangle$ is a model for a first-order language with two unary predicates P and Q and a binary relation predicate R. The domain of $\mathcal{M}$ is the set $\{a, b, c, d, e, f\}$, and the denotation of the predicates is:

- $\delta(P) = \{a, b, c\}$
- $\delta(Q) = \{d, e, f\}$
- $\delta(R) = \{\langle a, f \rangle, \langle b, e \rangle, \langle c, d \rangle, \langle f, f \rangle\}$

Which of the following formulae are satisfied by this model? **[4 marks]**

F1. $\forall x[P(x) \lor Q(x)]$

F2. $\exists w[P(w) \land Q(w)]$

F3. $\forall x[P(x) \rightarrow \exists y[R(x, y) \land Q(y)]]$

F4. $\neg\exists x \exists y[Q(x) \land Q(y) \land R(x, y)]$

Answer: F1 yes, F2 no, F3 yes, F4 no. 1 mark each.

(d) Use the *Sequent Calculus* to show that the following sequent is valid: **[6 marks]**

$$\forall x[P(x)],\ \ \forall x[P(x) \rightarrow Q(x)] \quad \vdash \quad \forall x[Q(x)]$$

You should only use rules from the following rule set, which was presented in the lecture slides, to construct your proof:

$$\frac{Axiom}{\alpha,\ \Gamma\ \vdash\ \alpha,\ \Delta}$$

$$\frac{\alpha,\ \beta,\ \Gamma\ \vdash\ \Delta}{(\alpha \wedge \beta),\ \Gamma\ \vdash\ \Delta}[\wedge\vdash] \qquad \frac{\Gamma \vdash\ \alpha,\ \Delta \quad \textit{and} \quad \Gamma \vdash \beta, \Delta}{\Gamma\ \vdash (\alpha \wedge \beta),\ \Delta}[\vdash\wedge]$$

$$\frac{\alpha, \Gamma\ \vdash\ \Delta \quad \textit{and} \quad \beta, \Gamma\ \vdash\ \Delta}{(\alpha \vee \beta),\ \Gamma\ \vdash\ \Delta}[\vee\vdash] \qquad \frac{\Gamma\ \vdash\ \alpha,\ \beta,\ \Delta}{\Gamma\ \vdash\ (\alpha\ \vee\ \beta),\ \Delta}[\vdash\ \vee]$$

$$\frac{\Gamma\ \vdash\ \alpha,\ \Delta}{\neg\alpha,\ \Gamma\ \vdash\ \Delta}[\neg\vdash] \qquad \frac{\Gamma,\ \alpha\ \vdash\ \Delta}{\Gamma\ \vdash\ \neg\alpha,\ \Delta}[\vdash\neg]$$

$$\frac{\Gamma,\ \neg\alpha \vee \beta\ \vdash\ \alpha,\ \Delta}{\Gamma,\ \alpha\ \rightarrow\ \beta\ \vdash\ \Delta}[\rightarrow\vdash r.w.] \qquad \frac{\Gamma\ \ \vdash\ \neg\alpha \vee \beta,\ \Delta}{\Gamma\ \vdash\ \alpha\ \rightarrow\ \beta,\ \Delta}[\vdash\rightarrow r.w.]$$

$$\frac{\forall x[\Phi(x)],\ \Phi(k),\ \Gamma\ \vdash\ \Delta}{\forall x[\Phi(x)],\ \Gamma\ \vdash\ \Delta}[\forall\vdash] \qquad \frac{\Gamma\ \vdash\ \Phi(k), \Delta}{\Gamma\ \vdash\ \forall x[\Phi(x)],\ \Delta}[\vdash\forall]^{\dagger}$$

† where κ cannot occur anywhere in the lower sequent.

Answer: The sequent is valid as shown by the following proof:

```
               AXIOM
----------------------------------------
Ax[Px], Pa,   Ax[Px -> Qx],  |-  Pa, Qa                          AXIOM
----------------------------------------[- |-]    ------------------------------------
Ax[Px], Pa,   Ax[Px -> Qx],  -Pa |-  Qa           Ax[Px], Pa, Ax[Px -> Qx], Qa  |-  Qa
--------------------------------------------------------------------------------------[v|-]
        Ax[Px],  Pa,   Ax[Px -> Qx],  -Pa v Qa  |-  Qa
        -------------------------------------------------[-> |- r.w.]
        Ax[Px],  Pa,   Ax[Px -> Qx],  Pa -> Qa  |-  Qa
        -------------------------------------------------[A |-]
        Ax[Px],  Pa,   Ax[Px -> Qx]  |-  Qa
        -------------------------------------[A |-]
        Ax[Px],  Ax[Px -> Qx]  |-  Qa
        -----------------------------------[|- A]
        Ax[Px],  Ax[Px -> Qx]  |-  Ax[Qx]
```

There is basically a mark for each correct rule application. Some credit may be given for almost correct rule applications.

[Question 1 total: 20 marks]

Question 2

(a) (i) Give the set of *clausal* formulae (i.e. formulae in *disjunctive normal form*) corresponding to the following propositional formulae: **[4 marks]**

$$\neg\neg A \vee S,\ (\neg S \wedge T),\ (A \vee B) \rightarrow Q,\ (Q \wedge T) \rightarrow (R \wedge S)$$

Answer:

$$\{\ \{A, S\},\ \{\neg S\},\ \{T\},\ \{\neg A, Q\},\ \{\neg B, Q\},\ \{\neg Q, \neg T, R\},\ \{\neg Q, \neg T, S\}\}$$

(ii) Give a proof that these formulae are inconsistent using *binary propositional resolution*. **[4 marks]**

Answer:

1. $\{A, S\}$	8.	$\{A\}$	1&2
2. $\{\neg S\}$	9.	$\{Q\}$	4&8
3. $\{T\}$	10.	$\{\neg T, S\}$	7&9
4. $\{\neg A, Q\}$	11.	$\{\neg T\}$	2&10
5. $\{\neg B, Q\}$	12.	$\emptyset$	3&11
6. $\{\neg Q, \neg T, R\}$			
7. $\{\neg Q, \neg T, S\}$			

(b) Translate the following sentence into *Propositional Tense Logic*: **[2 marks]**

If I win the lottery I will be rich forever after that.

Answer: $\mathbf{G}(\ \mathit{Win} \rightarrow \mathbf{G}\mathit{Rich}\ )$ (or $\mathbf{G}(\ \mathbf{P}\mathit{Win} \rightarrow \mathit{Rich}\ )$, though not quite right)

(c) A Situation Calculus theory makes use of fluents of the forms:

$robot\_has(item)$ $on\_floor(item, room)$ $locked(door)$
$robot\_location(room)$ $connects(door, room_1, room_2)$

The theory includes constants referring to items, one of which is key.

The theory also describes the behaviour of a robot in terms of the following actions:

$$\mathbf{pick\_up}(object)\quad \mathbf{unlock}(door)\quad \mathbf{move\_to}(room)$$

An initial situation, s_0, is described as follows:

$\mathsf{Holds}(connects(\mathsf{door1}, \mathsf{hall}, \mathsf{lounge}), s_0)$ $\mathsf{Holds}(connects(\mathsf{door2}, \mathsf{hall}, \mathsf{study}), s_0)$
$\neg\mathsf{Holds}(locked(\mathsf{door1}), s_0)$ $\mathsf{Holds}(locked(\mathsf{door2}), s_0)$
$\mathsf{Holds}(on\_floor(\mathsf{key}, \mathsf{lounge}), s_0)$ $\mathsf{Holds}(robot\_location(\mathsf{hall}), s_0)$

(i) Assuming that the initial situation is s_o, give a sequence of actions that will result in the goal $robot\_location(\mathsf{study})$ being satisfied. **[2 marks]**

(ii) For each of the actions $\mathbf{pick\_up}$ and $\mathbf{move\_to}$ specify a *precondition* axiom stating the conditions under which the action is possible. **[4 marks]**

(iii) Give an *effect* axiom specifying the results of carrying out the action $\mathbf{unlock}$. **[2 marks]**

(iv) Write down a *frame* axiom stating that the **move_to** action does not affect the *locked* fluent.

[2 marks]

Answer:

(i) *move(lounge), pickup(key), move(hall), unlock(door2), move(study)*

(ii)
- `Poss(pickup(x),s) <- E r[ Holds(robot_location(r),s) & Holds(Onfloor(x,r),s)]`
- `Poss(move(x),s) <- E r d[ Holds(connects(d,r,x),s) & Holds(robot_location(r),s) & - Holds(locked(d), s)]`

.

(It is actually also ok to leave out the existential quantifier in these axioms, since the usual implicit universal quantification converts to existential when on right of <-.)

(iii)
- `- Holds( locked(x), result(unlock(x),s)) <- Poss(unlock(x),s)`

(v) `Holds(locked(x), result(move(x),s)) <-> Holds(locked(x),s)`

[Question 2 total: 20 marks]

Question 3

(a) For each of the following *Prolog* queries, give the value of the variable `X` after the query has been executed:

(i) `?- X = 7/2.` **[1 mark]**

(ii) `?- [1, [2, 3], 4] = [ _ | [X | _ ]]` **[1 mark]**

(iii) `?- A = [1,2,3,4,5], setof( I, (member(I,A), I>2), X).` **[1 mark]**

(iv) `?- append( [X], [2,3], [1,2,3] ).` **[1 mark]**

Answer:

```
(i) 7/2
(ii) [2,3]
(iii) [3,4,5]
(iv) 1
```

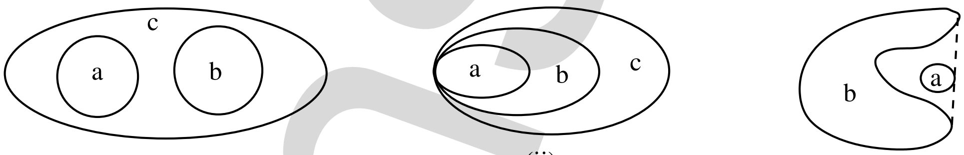
(b) Consider the following formulae involving topological relations of the *Region Connection Calculus* (RCC) and the *convex hull* function, conv. The constants (a, b and c) refer to particular spatial regions. In each case, draw a configuration of the regions referred to by these constants that satisfies the formula, labelling your diagram to indicate which region is which:

(i) $\mathsf{DC}(a, b) \wedge \mathsf{NTPP}(\mathsf{sum}(a, b), c)$ **[2 marks]**

(ii) $\mathsf{TPP}(a, b) \wedge \mathsf{TPP}(b, c) \wedge \mathsf{TPP}(a, c)$ **[2 marks]**

(iii) $\mathsf{DC}(a, b) \wedge \mathsf{TPP}(a, \mathsf{conv}(b))$ **[2 marks]**

Answer: Possible diagrams are as follows:



(i) (ii) (iii)

(c) A *liger* is an animal whose parents are a male lion and a female tiger. Use *Description Logic* to give a definition of the concept **Liger** in terms of the concepts **Lion**, **Tiger**, **Male**, **Female** and the relation **hasParent**. **[4 marks]**

Answer:

$$\mathsf{Liger} \equiv \exists \mathsf{hasParent}.(\mathsf{Male} \sqcap \mathsf{Lion}) \sqcap \exists \mathsf{hasParent}.(\mathsf{Female} \sqcap \mathsf{Tiger})$$

(d) Write a *Default Logic* rule that formally represents the reasoning principle expressed in the following statement: **[2 marks]**

"British people typically drink tea, except for children and those who drink coffee."

Answer: $British(x) : Drinks(x, tea), \neg Child(x), \neg Drinks(x, coffee) \;/ Drink(x, tea)$

(e) This question concerns a *Fuzzy Logic* in which the following definitions of *linguistic modifiers* are specified:

$$\mathsf{quite}(\phi) = \phi^{1/2} \qquad \mathsf{very}(\phi) = \phi^2$$

The logic is used to describe Leo the lion, who possesses certain characteristics to the following degrees:

$$\mathsf{Large}(\mathsf{leo}) = 0.5 \qquad \mathsf{Fierce}(\mathsf{leo}) = 0.09 \qquad \mathsf{Clever}(\mathsf{leo}) = 0.4$$

Translate the following sentences into fuzzy logic and also give the fuzzy truth value of each proposition (under the standard fuzzy interpretation of the Boolean connectives):

(i) Leo is not very clever. **[2 marks]**

(ii) Leo is very very large and quite fierce. **[2 marks]**

Answer:

A) $\neg\mathsf{very}(\mathsf{Clever}(\mathsf{leo}))$ (1 mark)

Truth value $1 - (0.4)^2 = 1 - 0.16 = 0.84$ (1 mark)

B) $\mathsf{very}(\mathsf{very}(\mathsf{Large}(\mathsf{leo}))) \wedge \mathsf{quite}(\mathsf{Fierce}(\mathsf{leo})))$ (1 mark)

Truth value = $Min(((0.5)^2)^2, (0.09)^{\frac{1}{2}}) = Min(0.0625, 0.3) = 0.0625$ (1 mark)

[Question 3 total: 20 marks]

[Grand total: 60 marks]